Exercise 36

Problems 35 and 36 require some knowledge of chemical notation.

(a) Write the chemical equation $p\mathrm{C_3H_4O_3} + q\mathrm{O_2} = r\mathrm{CO_2} + s\mathrm{H_2O}$ as an equation in ordered triples with unknown coefficients p, q, r, and s.

(b) Find the smallest positive integer solution for p, q, r, and s.

(c) Illustrate the solution by a vector diagram in space.

Solution

Let the ordered triplet (x_1, x_2, x_3) represent the number of carbon, hydrogen, and oxygen atoms, respectively.

$$p\mathrm{C_3H_4O_3} + q\mathrm{O_2} = r\mathrm{CO_2} + s\mathrm{H_2O}$$

$$p(3,4,3) + q(0,0,2) = r(1,0,2) + s(0,2,1)$$

$$(3p, 4p, 3p) + (0,0,2q) = (r,0,2r) + (0,2s,s)$$

$$(3p, 4p, 3p+2q) = (r, 2s, 2r+s)$$

Match the vector components.

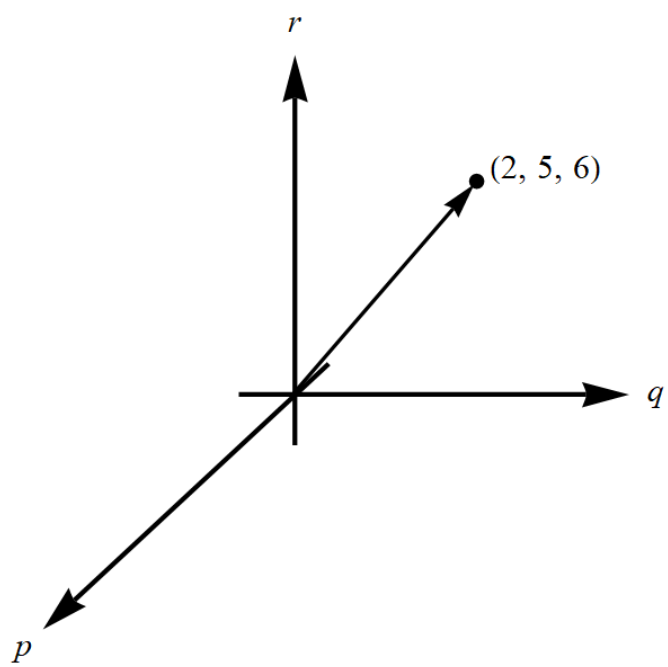
$$\begin{aligned} 3p &= r \\ 4p &= 2s \\ 3p + 2q &= 2r + s \end{aligned}$$

Solving this system of equations yields

$$p = \frac{s}{2} \quad \text{and} \quad q = \frac{5s}{4} \quad \text{and} \quad r = \frac{3s}{2}.$$

In order for all the coefficients to be the smallest integers, set $s = 4$. Then

$$p = 2 \quad \text{and} \quad q = 5 \quad \text{and} \quad r = 6.$$


r
(2, 5, 6)
q
p